<u>**Chapter 4 Factors and Polynomials**</u>

1. $p(x) = ax^3 + 3x^2 + bx - 12$ has a factor of $2x + 1$. When $p(x)$ is divided by $x - 3$ the remainder is 105.

   a. Find the value of *a* and of *b*. [5]

$$p(-\tfrac{1}{2}) = -\frac{a}{8} + \frac{3}{4} - \frac{b}{2} - 12$$

$$0 = -a + 6 - 4b - 96$$

$$0 = -a - 4b - 90$$

$$a + 4b = -90 \quad — ① \times 3$$

$$p(3) = 27a + 27 + 3b - 12$$

$$105 = 27a + 3b + 15$$

$$27a + 3b = 90 \quad — ② \times 4$$

$$3a + \cancel{12b} = -270$$

$$-108a + -\cancel{12b} = -360$$

$$105a = 630$$

$$a = 6$$

$$6 + 4b = -90$$

$$4b = -96$$

$$b = -24$$

   b. Using your values of *a* and *b*, write p (*x*) as a product of $2x + 1$ and a quadratic factor. [2]

$$\begin{array}{r|l} & 3x^2 - 12 \\ \hline 2x+1 & 6x^3 + 3x^2 - 24x - 12 \\ & -6x^3 + -3x^2 \\ \hline & \quad -24x - 12 \\ & \quad -24x - 12 \\ \hline & \quad 0 \end{array}$$

$$p(x) = (3x^2 - 12)(2x + 1)$$

   c. Hence solve $p(x) = 0$. [2]

$$p(x) = (3x^2 - 12)(2x + 1)$$

$$3x^2 - 12 = 0 \qquad 2x + 1 = 0$$

$$3x^2 = 12 \qquad 2x = -1$$

$$x^2 = 4 \qquad x = -\frac{1}{2}$$

$$x = \pm 2$$

2. $p(x) = 6x^3 + ax^2 + 12x + b$, where *a* and *b* are integers. $p(x)$ has a remainder of 11 when divided by $x - 3$ and a remainder of -21 when divided by $x + 1$.

(a) Given that $p(x) = (x - 2)Q(x)$,find Q(*x*), a quadratic factor with numerical coefficients.

[6]

$p(x) = 6x^3 + ax^2 + 12x + b$

$p(3) = 11$

$162 + 9a + 36 + b = 11$

$9a + b = -187$

$p(-1) = -21$

$-6 + a - 12 + b = -21$

$a + \cancel{b} = -3$

$-9a - \cancel{b} = +187$

$-8a = 184$

$a = -23$

$b = -3 - a$

$= -3 + 23$

$= 20$

$p(x) = 6x^3 - 23x^2 + 12x + 20$

$$\begin{array}{r|l} & 6x^2 - 11x - 10 \\ \hline x-2 & 6x^3 - 23x^2 + 12x + 20 \\ & -(6x^3 - 12x^2) \\ \hline & -11x^2 + 12x \\ & -(-11x^2 + 22x) \\ \hline & -10x + 20 \\ & -10x + 20 \\ \hline & 0 \end{array}$$

$Q(x) = 6x^2 - 11x - 10$

(b) Hence solve $p(x) = 0$.

[2]

$(x-2)(6x^2 - 11x - 10) = 0$

$x = 2$ or $(2x-5)(3x+2) = 0$

$x = \frac{5}{2}$ or $x = -\frac{2}{3}$

3. **DO NOT USE A CALCULATOR IN THIS QUESTION.**

$$p(x) = 15x^3 + 22x^2 - 15x + 2$$

(a) Find the remainder when p(x) is divided by x + 1. [2]

$p(-1) = -\cancel{15} + 22 + \cancel{15} + 2$

$= 24$

(b) (i) Show that x+2 is a factor of p(x). [1]

$p(x) = 15x^3 + 22x^2 - 15x + 2$

$p(-2) = -120 + 88 + 30 + 2$

$= 0$

$\therefore$ $x+2$ is a factor of $p(x)$.

(ii) Write p(x) as a product of linear factors. [3]

$15x^2 - 8x + 1$

$x+2 \,\overline{)\, 15x^3 + 22x^2 - 15x + 2}$

$ \overset{-}{} 15x^3 \overset{-}{+} 30x^2$

$-8x^2 - 15x$

$\overset{+}{-}8x^2 \overset{+}{-} 16x$

$x + 2$

$x + 2$

0

$p(x) = (x+2)(15x^2 - 8x + 1)$

$(x+2)(3x-1)(5x-1)$

4. The polynomial $p(x) = 6x^3 + ax^2 + bx + 2$, where *a* and *b* are integers, has a factor of $x - 2$.

a. Given that $p(1) = -2p(0)$, find the value of *a* and of *b*. [4]

$p(1) = 6 + a + b + 2$
$= a + b + 8$

$p(0) = 2$
$p(1) = -2p(0)$
$a + b + 8 = -4$
$a + b = -12$ — ①
$p(2) = 48 + 4a + 2b + 2$
$4a + \cancel{2b} = -50$ — ②
$2a + \cancel{2b} = -24$ — ①
$2a = -26$
$a = -13$
$b = -12 - a$
$= -12 + 13$
$= 1$

b. Using your values of *a* and *b*,

i. find the remainder when p (*x*) is divided by $2x - 1$, [2]

$p(x) = 6x^3 - 13x^2 + x + 2$
$p(\frac{1}{2}) = \frac{3}{4} - \frac{13}{4} + \frac{1}{2} + 2$
$= 0$

ii. factorise p (*x*) . [2]

$$
\begin{array}{r|l}
 & 3x^2 - 5x - 2 \\
2x - 1 & 6x^3 - 13x^2 + x + 2 \\
 & 6x^3 - 3x^2 \\
 & -10x^2 + x \\
 & -10x^2 + 5x \\
 & -4x + 2 \\
 & -4x + 2 \\
 & 0
\end{array}
$$

$p(x) = (2x-1)(3x^2 - 5x - 2)$
$= (2x-1)(x-2)(3x+1)$

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5. The polynomial $p(x) = ax^3 + bx^2 - 19x + 4$, where *a* and *b* are constants, has a factor $x + 4$ and is such that $2p(1) = 5p(0)$.

a. Show that $p(x) = (x + 4)(Ax^2 + Bx + C)$, where *A*, *B* and *C* are integers to be found. [6]

$p(x) = ax^3 + bx^2 - 19x + 4$

$p(-4) = -64a + 16b + 76 + 4$

$0 = -64a + 16b + 80$

$64a - 16b = 80$

$4a - b = 5$ — ①

$p(1) = a + b - 19 + 4$

$= a + b - 15$

$2p(1) = 2a + 2b - 30$

$5p(0) = 4 \times 5 = 20$

$2a + 2b = 50$

$a + b = 25$ — ②

$4a - b = 5$

$5a = 30$

$a = 6$

$b = 25 - a$

$= 25 - 6$

$= 19$

$p(x) = 6x^3 + 19x^2 - 19x + 4$

$$\begin{array}{r|l} & 6x^2 - 5x + 1 \\ \hline x+4 & 6x^3 + 19x^2 - 19x + 4 \\ & -(6x^3 + 24x^2) \\ \hline & -5x^2 - 19x \\ & -5x^2 - 20x \\ \hline & x + 4 \\ & x + 4 \\ \hline & 0 \end{array}$$

$p(x) = (x+4)(6x^2 - 5x + 1)$

b. Hence factorise $p(x)$. [1]

$p(x) = (x+4)(6x^2 - 5x + 1)$

$= (x+4)(2x-1)(3x-1)$

c. Find the remainder when $p'(x)$ is divided by x. **(Chapter 12)** [1]

$p'(x) = 18x^2 + 38x - 19$

$R = -19$

6. **DO NOT USE A CALCULATOR IN THIS QUESTION.**

$$p(x) = 2x^3 - 3x^2 - 23x + 12$$

a. Find the value of $p(\frac{1}{2})$. [1]

$p(\frac{1}{2}) = \frac{2}{8} - \frac{3}{4} - \frac{23}{2} + 12$

$= 0$

b. Write p(x) as the product of three linear factors and hence solve p(x) = 0. [5]

$$
\begin{array}{r|l}
 & x^2 - x - 12 \\
2x-1 & 2x^3 - 3x^2 - 23x + 12 \\
 & -(2x^3 - x^2) \\
\hline
 & -2x^2 - 23x \\
 & -(-2x^2 + x) \\
\hline
 & -24x + 12 \\
 & -24x + 12 \\
\hline
 & 0
\end{array}
$$

$p(x) = (2x-1)(x^2-x-12)$

$= (2x-1)(x-4)(x+3)$

$p(x) = 0$

$x = \frac{1}{2}$ or $x = 4$ or $x = -3$